

All answers should include your work (this could be a written explanation of the result, a graph with the relevant feature pointed out or an algebraic solution). On the Final Exam, less work = fewer points!!

Determine whether or not the relationship shown is a function. Explain why or why not.

1)

January	1	2	3	4	5	6	7
Weight (lbs)	210	209	211	210	209	208	207

Does the table define weight as a function of the day in January? Why or why not?

Solve the problem.

- 2) Suppose a cost-benefit model is given by  $C(p) = \frac{1.5p}{100 - p}$ , where C is the cost in thousands of dollars for removing p percent of a given pollutant. Find C(85) to the nearest dollar and interpret it.
- 3) Suppose the sales of a particular brand of appliance are modeled by the linear function  $S(x) = 50x + 5800$ , where S(x) represents the number of sales in year x, with x = 0 corresponding to 1982. Find the number of sales in 1996.
- 4) The population of a small U.S. town can be modeled by  $P = -37t + 13,500$ , where t is the number of years since 1975. Find and interpret all intercepts of the graph of this function.
- 5) A boat is moving away from shore in such a way that at time t hours its distance from shore, in kilometers, is given by the linear function  $d(t) = 4.5t + 8.7$ . What is the rate of change of the distance from shore?

Find the zero of f.

6)  $f(x) = 4x + 8$

Solve the formula for the specified variable.

7)  $F = \frac{9}{5}C + 32$  for C

Solve the problem.

- 8) A gas station sells 4820 gallons of regular unleaded gasoline on a day when they charge \$1.35 per gallon, whereas they sell 3917 gallons on a day that they charge \$1.40 per gallon. Find a linear function that expresses gallons sold as a function of price. (Let x = selling price)
- 9) An average score of 90 for 5 exams is needed for a final grade of A. John's first 4 exam grades are 79, 89, 97, and 95. Determine the grade needed on the fifth exam to get an A in the course.

Use the table to determine whether the data set represented is exactly linear, approximately linear, or nonlinear.

10)

x	1	2	3	4	5
y	9	11	14	18	23

Write the best-fit linear model for the data.

- 11) A pediatric speech therapist started her own practice in 1990. The table below shows the number of children she treated each year from 1990 to 1996. Align the data to the number of years past 1990 and fit a linear model to the data.

Year	1990	1991	1992	1993	1994	1995	1996
# of Patients	11	15	16	20	23	25	29

Solve the problem.

- 12) A shoe company will make a new type of shoe. The fixed cost for the production will be \$24,000. The variable cost will be \$37 per pair of shoes. The shoes will sell for \$103 for each pair. What is the profit if 600 pairs are sold?

13) A toilet manufacturer has decided to come out with a new and improved toilet. The fixed cost for the production of this new toilet line is \$16,600 and the variable costs are \$63 per toilet. The company expects to sell the toilets for \$150. Formulate a function  $C(x)$  for the total cost of producing  $x$  new toilets and a function  $R(x)$  for the total revenue generated from the sales of  $x$  toilets.

14) At Allied Electronics, production has begun on the X-15 Computer Chip. The total revenue function is given by  $R(x) = 43x - 0.3x^2$  and the total cost function is given by  $C(x) = 10x + 12$ , where  $x$  represents the number of boxes of computer chips produced. Find  $P(x)$ .

15) Midtown Delivery Service delivers packages which cost \$1.70 per package to deliver. The fixed cost to run the delivery truck is \$80 per day. If the company charges \$6.70 per package, how many packages must be delivered daily to break even?

16) Find the market equilibrium point for the following supply and demand functions and explain what it means.  
 $D(p) = 790 - 12p$ ;  
 $S(p) = 390 + 3p$

17) Suppose that the sales of a particular brand of appliance satisfy the relationship  $S(x) = 140x + 1900$ , where  $S(x)$  represents the number of sales in year  $x$ , with  $x = 0$  corresponding to 1990. For what years will sales be between 2740 and 3160?

18) John owns a hot dog stand. He has found that his profit is given by the equation  $P = -x^2 + 66x + 73$ , where  $x$  is the number of hot dogs sold. How many hot dogs must he sell to earn the most profit?

**Scale the axes and sketch the graph listing the vertex and all intercepts.**

$$21) y = 60x^2 + 171x - 1275$$

**Find the x-intercepts.**

$$22) y = x^2 - 3x - 18$$

**Use the quadratic formula to solve the equation and give the solution in simplest radical form (exact answers).**

$$23) 7m^2 + 10m + 2 = 0$$

**Solve the problem.**

24) The demand equation for a certain product is  $P = 35 - 0.002x$ , where  $x$  is the number of units sold per week and  $P$  is the price in dollars at which one is sold. The weekly revenue  $R$  is given by  $R = xP$ . What number of units sold produces a weekly revenue of \$70,000?

**Determine if the function is increasing or decreasing over the interval indicated.**

$$25) f(x) = \sqrt{4 - x}; \quad \text{interval: } (-, 4)$$

**Write the equation of the graph after the indicated transformation(s).**

26) The graph of  $y = x^2$  is shifted 2 units to the left. This graph is then vertically stretched by a factor of 5 and reflected across the  $x$ -axis. Finally, the graph is shifted 8 units downward.

**Solve the equation algebraically.**

$$27) \sqrt{4x - 3} = 2x - 3$$

**Solve the problem.**

28) A study in a small town showed that the percent of residents who have college degrees can be modeled by the function  $P = 35x^{0.35}$ , where  $x$  is the number of years since 1990. According to the model, what percent of residents had a college degree in 1995? Round to the nearest whole number.

**Find the coordinates of the vertex and determine whether the graph is concave up or down.**

$$19) y = (x + 8)^2 - 3$$

$$20) y = -4(x - 3)^2 - 4$$

- 29) A furniture manufacturer decides to make a new line of desks. The table shows the profit, in thousands of dollars, for various levels of production.

Number of Desks Produced	120	350	500	650	750
Profit (Thousands)	13	37	44	34	25

Find a quadratic function to model the data, and use the model to predict the profit if 450 desks are made.

- 30) The percent of people who say they plan to stay in the same job position until they retire has decreased over recent years, as shown in the table below.

Year	1995	1996	1997	1998	1999	2000
Percent	42	38	35	34	30	26

Find a power function that models the data in the table using an input equal to the number of years from 1990.

**Graph the function.**

$$31) f(x) = \begin{cases} x - 2 & \text{for } x > 0 \\ -1 & \text{for } x \leq 0 \end{cases}$$

- 32) Determine whether a linear or quadratic function would be a more appropriate model for the graphed data. If linear, tell whether the slope should be positive or negative. If quadratic, decide whether the coefficient of  $x^2$  should be positive or negative.

**Dental Floss Use:**

Years since 1990	1	2	4	6	8
People (millions)	3.2	5.2	7.1	5.2	0.7

**Find the requested value.**

- 33) A married couple's federal income tax in 2002 could be determined using the following function where  $x$  is the taxable income up to \$109,250:

$$f(x) =$$

$$\begin{cases} 0.10x, & \text{if } x \leq 12000 \\ 1200 + 0.15(x-12000), & \text{if } 12000 < x \leq 45200 \\ 5280 + 0.28(x-45200), & \text{if } 45200 < x \leq 109250 \end{cases}$$

What would the couple's income tax be if their taxable income was \$35,000.

**For problems 34-51 find the new function and the**

$$34) f(x) = \sqrt{x-7} \text{ and } g(x) = \frac{1}{x-8}$$

Find  $(f \cdot g)(x)$  and its domain.

$$35) \text{ Given } f(x) = \frac{7}{x-6} \text{ and } g(x) = \frac{5}{7x},$$

Find  $(f \cdot g)(x)$  and its domain.

$$36) \text{ For } f(x) = x^2 - 36 \text{ and } g(x) = 2x + 3$$

Find  $\left(\frac{f}{g}\right)(x)$  and its domain.

**Solve the problem.**

- 37) The cost of manufacturing clocks is given by  $C(x) = 67 + 40x - x^2$ . Also, it is known that in  $t$  hours the number of clocks that can be produced is given by  $x = 8t$ , where  $1 \leq t \leq 12$ . Express  $C$  as a function of  $t$ .

**Determine whether or not the function is one-to-one and explain why or why not.**

- 38) This chart shows the number of hits for five Little League baseball teams.

Team	Hits
Hawks	28
Lions	41
Eagles	55
Bears	55
Dolphins	20

$$39) f(x) = 4x^2 + x$$

**Find the inverse of the function.**

40)  $f(x) = x^3 - 3$

**Use algebraic and graphical or numerical methods to solve the inequality.**

41)  $x^2 + 2x < 3$

**Solve the problem.**

42) The cost  $C$  of producing  $t$  units is given by  $C(t) = 4t^2 + 9t$ , and the revenue  $R$  generated from selling  $t$  units is given by  $R(t) = 5t^2 + t$ . For what values of  $t$  will there be a profit?

43) A coin is tossed upward from a balcony 262 ft high ( $h_0$ ) with an initial velocity ( $v_0$ ) of 16 ft/sec, according to the formula  $h(t) = -16t^2 + v_0t + h_0$ , where  $t$  is time in seconds. During what interval of time will the coin be at a height of at least 70 ft?

44) The number of books in a small library increases according to the function  $B = 3600e^{0.05t}$ , where  $t$  is measured in years after opening.

a) How many books did the library have when they opened?

b) How many books will the library have 6 years after opening?

**Graph the function showing all asymptotes and**

45)  $f(x) = 4^{(x+1)} - 2$

46)  $f(x) = \log_4(x + 2)$

**Provide an appropriate response.**

47) Explain how the graph of  $y = \left(\frac{1}{3}\right)^{(x-4)} + 5$  can be obtained from the graph of  $y = 3^x$ .

48) Explain how the graph of  $y = -5\log_2(x+3) - 1$  can be obtained from the graph of  $y = \log_2x$ .

**Write the logarithmic equation in exponential form.**

49)  $\log_4 \frac{1}{64} = -3$

50)  $\ln x = -6$

**Write the exponential equation in logarithmic form.**

51)  $6^3 = 216$

52)  $e^{4x} = y$

**Find the value of the logarithm without using a calculator.**

53)  $\log_5 \sqrt{5}$

**Use a change of base formula to evaluate the given logarithm. Approximate to three decimal places.**

54)  $\log_4(0.654)$

**Find the inverse of the function.**

55)  $f(x) = 2^{-x} + 3$

**Solve the problem.**

56) The logarithmic function  $f(x) = -100 + 87 \ln x$  models the number of visitors (in millions) to U.S. museums from 1940 to 1990, where  $x$  is the number of years since 1900. Use this function to estimate the number of visitors in the year 1983.

**Provide an appropriate response.**

57) For the function:  $f(x) = \log_{10} x$

a) What is the domain of the function?

b) What is the range of the function?

**Solve the equation algebraically.**

58)  $29^x = 89$  (Round to two decimal places.)

59)  $50e^{-0.647x} = 125$  (Round to three decimal places.)

60)  $\log 4x = \log 5 + \log(x - 4)$

61)  $\log_2(3x - 2) - \log_2(x - 5) = 4$

62)  $5 \ln(x - 4) = 10$

**Rewrite the expression as the sum and/or difference of logarithms, without using exponents. Simplify if**

63)  $\log_4 \frac{x^8 y^9}{3}$

**Rewrite as a single logarithm.**

64)  $(\log_t t - \log_t s) + 3 \log_t u$

**Solve the problem.**

65) Assume the cost of a car is \$16,000. With continuous compounding in effect, the cost of the car will increase according to the equation  $C = 16,000e^{rt}$ , where  $r$  is the annual inflation rate and  $t$  is the number of years. Find the number of years it would take to double the cost of the car at an annual inflation rate of 2.1%. Round the answer to the nearest hundredth.

66) In the formula  $A(t) = A_0 e^{kt}$ ,  $A$  is the amount of radioactive material remaining from an initial amount  $A_0$  at a given time  $t$ , and  $k$  is a negative constant determined by the nature of the material. A certain radioactive isotope decays at a rate of 0.175% annually. Determine the half-life of this isotope, to the nearest year.

67) A bacteria culture starts with 10,000 bacteria, and the number doubles every 40 minutes.

a) Find the exponential growth function that models the number of bacteria after  $t$  minutes.

b) Find the number of bacteria after one hour.

c) After how many minutes will there be 50,000 bacteria?

68) Find an exponential function that models the data below and use it to predict about how many books will have been read in the eighth grade.

Grade	Number of Books Read
2	9
3	27
4	67
5	121

69) A radioactive substance has a half-life of 140 days. Suppose a sample of this substance has a mass of 300 mg.

a) Find an exponential decay function that models the amount of the sample remaining after  $t$  days.

b) Find the mass remaining after one year.

c) How long will it take for the sample to decay to a mass of 200 mg?

70) Find a natural logarithmic regression equation for the following data and use it estimate the production level for the year 1985. (For the regression equation, assume  $t = 0$  is the year 1950.)

Year	Production (in millions)
1960	9.26
1970	51.43
1990	68.91

**Provide an appropriate response.**

71) Does it appear that a linear model or an exponential model is the better fit for the data given in the table below? Explain your choice.

x	y
2	6
4	8.5
6	10.8
8	13.3

**Solve the problem.**

72) Find the amount of money in an account after 9 years if \$3500 is deposited at 4% annual interest compounded monthly.

73) How long would it take \$9000 to grow to \$45,000 at 7% compounded continuously? Round your answer to the nearest tenth of a year.

74) Find the annual interest rate if \$3500 is deposited in an account that compounds semi-annually and after 1.5 years the future value is \$4000

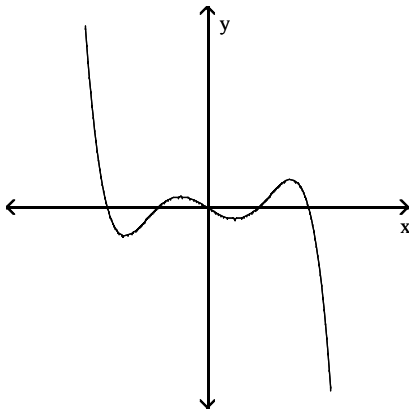
- 75) The number of students infected with the flu on a college campus after  $t$  days is modeled by the function  $P(t) = \frac{320}{1 + 39e^{-0.3t}}$ . What is the maximum number of infected students possible?

**Determine a window which gives a complete graph of the polynomial function, sketch the graph, and locate all intercepts and local maximum and minimum points.**

76)  $y = 5x^5 + x^3 - 4x^2 - 3x - 25$

**Determine the minimum degree of the polynomial and state whether the leading coefficient is positive or negative.**

77)



**Solve the problem.**

- 78) Suppose that the population of a certain city can be approximated by:  
 $P(x) = -0.1x^5 + 3.5x^4 + 5000$   
 where  $x$  is time in years since 1960. By sketching a graph of  $P(x)$ , estimate during what time period the population of the city was increasing.

- 79)  $S(x) = -x^3 + 6x^2 + 288x + 4000$ ,  $4 \leq x \leq 20$  is an approximation to the number of salmon swimming upstream to spawn, where  $x$  represents the water temperature in degrees Celsius. Find the temperature that produces the maximum number of salmon.

- 80) The table lists the percentage  $y$  of high school seniors that had used a certain drug within the previous month in a certain city for various years  $x$ . In this table  $x = 0$  corresponds to 1975.

$x$ (yr)	0	2	4	8	12	16	20	24
$y$ (%)	32	42	44	36	31	23	29	37

- a) Make a scatter plot of the data to determine whether a linear, quadratic, cubic, or quartic polynomial would best fit the data.  
 b) Use regression on your calculator to find the equation that best models the data.

**Solve the polynomial equation by using the root method.**

81)  $7x^3 - 189 = 0$

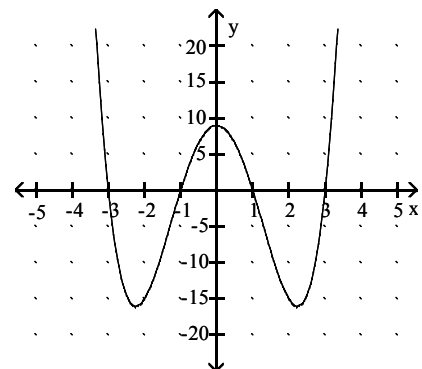
**Solve the polynomial equation by factoring.**

82)  $x^3 - 100x = 0$

83)  $2x^3 + 4x^2 - 2x - 4 = 0$

**Use the graph of the polynomial function  $f(x)$  to solve  $f(x) = 0$ .**

84)



**Solve the problem.**

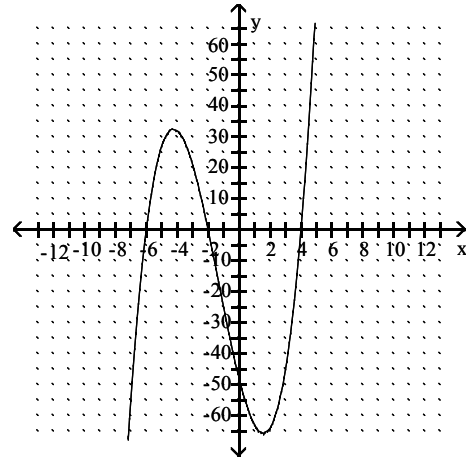
- 85) If the price for a product is given by  $p = 900 - x^2$ , where  $x$  is the number of units sold, then the revenue is given by  $R = px = 900x - x^3$ . How many units must be sold to give zero revenue?
- 86) Suppose a business can sell  $x$  gadgets for  $p = 250 - 0.01x$  dollars apiece, and it costs the business  $c(x) = 1000 + 25x$  dollars to produce the  $x$  gadgets. Determine the production level required to maximize profit.

- 87) The Cool Company determines that the supply function for its basic air conditioning unit is  $S(p) = 50 + 0.01p^3$  and that its demand function is  $D(p) = 250 - 0.2p^2$ , where  $p$  is the price. Determine the price for which the supply equals the demand.

95)  $\frac{2x + 4}{5x^2 - 5} > 0$

Use the graph of  $f$  to solve the inequality.

96)  $f(x) < 0$



Graph the function showing all asymptotes and

88)  $f(x) = \frac{x^2 + x - 4}{x^2 - 4}$

For the given rational function  $f$ , find all values of  $x$  for which  $f(x)$  has the indicated value.

89)  $f(x) = \frac{x - 2}{x - 4}$ ;  $f(x) = 3$

Use analytical methods to solve the equation.

90)  $\frac{40}{x - 2} = 1 + \frac{42}{x + 2}$

Solve the problem.

- 91) In the following formula,  $y$  is the minimum number of hours of studying required to attain a test score of  $x$ :  $y = \frac{0.38x}{100.5 - x}$ . How many hours of study are needed to score 90?

- 92) The percent of concentration of a certain drug in the bloodstream  $x$  hr after the drug is administered is given by  $K(x) = \frac{2x}{x^2 + 25}$ . At what time is the concentration a maximum?

- 93) Suppose that a cost-benefit model is given by

$$f(x) = \frac{5.9x}{100 - x}$$

where  $f(x)$  is the cost in thousands of dollars of removing  $x$  percent of a given pollutant. What is the vertical asymptote of the graph of this function? What does this suggest about the possibility of removing all of the pollutant? Explain your reasoning.

Solve the inequality.

94)  $(x + 9)(x + 5)(x - 8) > 0$

Solve the problem.

- 97) An open-top box is to be made by cutting small identical squares from each corner of a 12-by-12-in. sheet of tin and bending up the sides. If each corner square is  $x$  inches on a side, the volume of the box (in  $\text{in}^3$ ) is given by:

$$V(x) = 144x - 48x^2 + 4x^3$$

By sketching the graph of  $V(x)$ , estimate what values of  $x$  result in a box with a volume greater than  $64 \text{ in}^3$ .

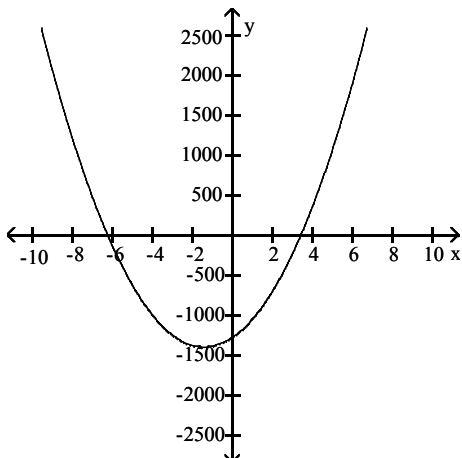
- 98) The monthly sales volume  $y$  (in thousands of dollars) is related to monthly advertising expenditures  $x$  (in thousands of dollars) according to the equation  $y = \frac{100x}{x + 10}$ .

Spending how much money on advertising will result in sales of at least \$75,000 per month?

# Answer Key

## Testname: M122 FINAL REVIEW

- 1) Yes. For each input value (day) there is exactly one output value (weight)
- 2) \$8500; It will cost \$8500 to remove 85% of the pollutant.
- 3) 6500
- 4) (0, 13500) : The population of the town was 13,500 in 1975.  
(364.86, 0): If the population continues to decrease at the same rate, the population will be zero after 365 years (2340).
- 5) 4.5 km/hr
- 6) -2
- 7)  $C = \frac{5}{9}(F - 32)$
- 8)  $G(p) = -18,060x + 29,201$
- 9) 90
- 10) Nonlinear
- 11)  $y = 2.89x + 11.18$
- 12) \$15,600
- 13)  $C(x) = 16600 + 63x$ ;  $R(x) = 150x$
- 14)  $P(x) = -0.3x^2 + 33x - 12$
- 15) 16 packages
- 16) When the price is \$26.67, the amount demanded equals the amount supplied which is 470.
- 17) Between 1996 and 1999
- 18) 33 hot dogs
- 19) (-8, -3); concave up
- 20) (3, -4) Concave down
- 21) vertex: (-1.425, -1396.8375)  
intercepts: (3.4, 0) (-6.25, 0) (0, -1275)



22) (6, 0) and (-3, 0)

23)  $\frac{-5 \pm \sqrt{11}}{7}$

24) 2303 or 15,197

25) Decreasing

26)  $y = -5(x + 2)^2 - 8$

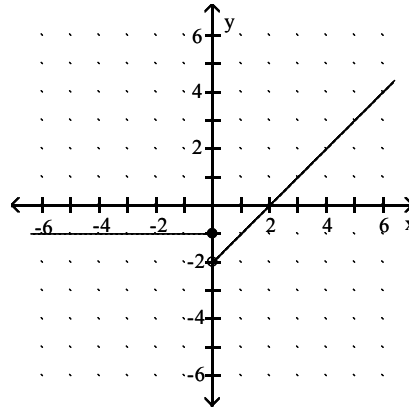
27) 3

28) 61%

29)  $y = -.000231778x^2 + .220750611x - 10.440468726$   
Almost \$42,000

30)  $y = 120.077x^{-0.638}$

31)



32) Quadratic; negative

33) \$4,650

34)  $\frac{\sqrt{x-7}}{x-8}$ ; domain: [7,8) (8, )

35)  $\frac{49x}{5-42x}$ ; domain:  $\{x \mid x \neq 0, x \neq \frac{5}{42}\}$

36)  $\frac{x^2-36}{2x+3}$ ;  $\{x \mid x \neq -\frac{3}{2}\}$

37)  $C(t) = 67 + 320t - 64t^2$

38) No, the output value of 55 is repeated with different input values.

39) No, its graph does not pass the horizontal line test.

40)  $f^{-1}(x) = \sqrt[3]{x+3}$

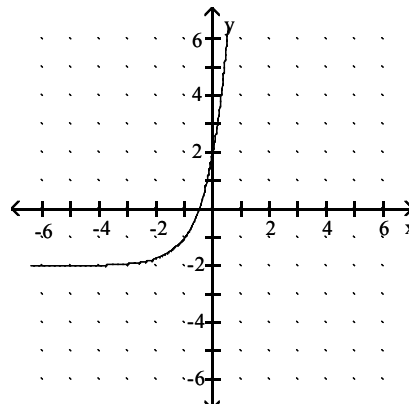
41)  $-3 \leq x \leq 1$

42)  $t > 8$

43) 0 sec  $t = 4$  sec

44) 3600; 4860

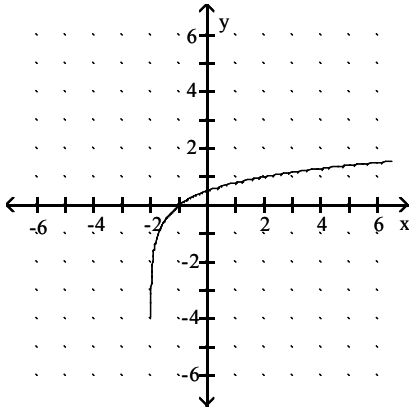
45)



# Answer Key

## Testname: M122 FINAL REVIEW

46)



47) The graph is reflected over the y-axis, shifted 4 units to the right and 5 units up.

48) The graph is shifted 3 units to the left, stretched by a factor of 5, reflected across the x-axis and shifted 1 unit down.

49)  $4^{-3} = \frac{1}{64}$

50)  $x = e^{-6}$

51)  $\log_6 216 = 3$

52)  $\ln y = 4x$

53)  $\frac{1}{2}$

54)  $-0.306$

55)  $f^{-1}(x) = -\log_2(x - 3)$

56) 284.4 million

57) a)  $(0, )$  b)  $(- , )$

58) 1.33

59)  $-1.416$

60)  $\{20\}$

61)  $x = 6$

62)  $e^2 + 4 = 11.39$

63)  $8 \log_4 x + 9 \log_4 y - \log_4 3$

64)  $\log_t \frac{tu^3}{s}$

65) 33.01

66) 396 yr

67) a)  $n(t) = 10000e^{0.01733t}$

b) 28,287 bacteria

c) 93 minutes

68)  $f(x) = 1.780319(2.387998)^x$   
1883 books

69) a)  $m(t) = 300e^{-0.00495t}$

b) 49 mg.

c) 82 days

70) 67.28

71) Since the first differences are close to being equal, a linear model will be a better fit than an exponential model.

72) \$5013.65

73) 23.0 years

74) 9.1%

75) 320

76)  $[-5, 5]$  by  $[-50, 50]$ ;

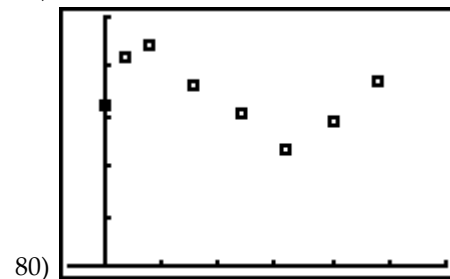
intercepts:  $(1.48, 0)$ ;  $(0, -25)$ ;

max:  $(-0.31, -24.5)$ , min:  $(0.73, -27.90)$

77) Minimum degree is 5. Leading coefficient is negative.

78) Between 1960 and 1988

79)  $12^\circ\text{C}$



80)

Cubic

$$f(x) = .015x^3 - .491x^2 + 3.502x + 34.443$$

81) 3

82) 0, 10, -10

83) -2, 1, -1

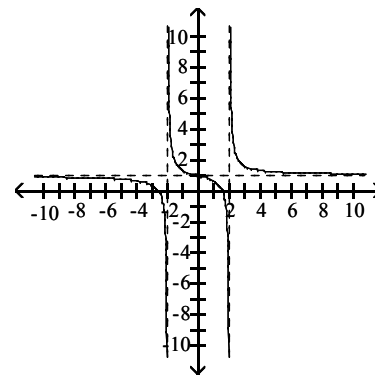
84) -3, -1, 1, 3

85) 0, 30

86) 11,250 gadgets

87) \$21.86

88)



89) 5

90) 12, -14

91) 3.26 hr

92) 5 hr

## Answer Key

Testname: M122 FINAL REVIEW

93) Answers will vary. Possible answer: The vertical asymptote is  $x = 100$ . This suggests that as  $x$  approaches 100 percent, the cost becomes infinitely large. It is therefore not possible to remove all of the pollutant.

94)  $-9 < x < -5$  or  $x > 8$

95)  $(-2, -1)$   $(1, )$

96)  $x < -6$  or  $-2 < x < 4$

97) 0.54 in.  $x$  4 in.

98) At least \$30,000