M119 Final Review – *Revised for 9th edition, Spring 08*

***Note: Be sure to review material not included on this review as well**

1.1 1. What is the domain of a) $f(x) = \frac{x^2-1}{x^2-9}$ b) $f(x) = \sqrt{3x-4}$

1.1 2. Find $g(h(x))$ where $g(u) = u^2 - u$, $h(x) = \frac{x}{x+1}$

1.1 3. At a certain factory, the total cost of manufacturing q units during the daily production run is $C(q) = q^2 + q + 900$ dollars. On a typical workday, $q(t) = 25t$ units are manufactured during the first t hours of a production run. (a) Express the total manufacturing cost as a function of t. (b) How much will have been spent on production by the end of the third hour? (c) When will the total manufacturing cost reach $11,000?

1.2 4. Sketch and label the graph of $f(x) = \begin{cases} x^2 & \text{if } x<1 \\ 3 & \text{if } x=1 \\ 1-x^2 & \text{if } x>1 \end{cases}$

1.2 5. Find the points of intersection of the following functions: $y = x^3 - 8x$ and $y = x$

1.3 6. Since the beginning of the month, a local reservoir has been losing water at a constant rate. On the 10th of the month the reservoir held 180 million gallons of water, and on the 20th day it held only 168 million gallons. (a) Express the amount of water in the reservoir as a function of the time.
(b) How much water was in the reservoir on the 7th day of the month?

1.3 7. During the winter, a group of students builds sleds in a converted garage. The rental for the garage is $550 for the winter, and the material needed to build a sled cost $15. The sleds can be sold for $70 apiece. How many sleds must be sold to break even?

1.4 8. The supply function for the sale of a product at p dollars a unit is $S(p) = p - 8$ and the demand function is $D(p) = \frac{4340}{p}$. Find the equilibrium price.

1.4, 3.5 9. An open box with a square base and vertical sides is constructed out of 300 cm^2 of tin.
a) Express the volume of the box as a function of x, an edge of its base.
b) Use calculus to find the value of x which maximizes the volume and the maximum volume.

2.2 10. Find the equation of the line tangent to the curve $y = x^3 - x$ at the point where $x = -1$.

2.2, 2.1 11. Find where the graph of $f(x) = 4 - 2x^2$ has a horizontal tangent line.

2.2 12. Differentiate and Simplify: $f(x) = x^3 - \frac{2}{x} + 2x^{3/2} - \frac{5}{\sqrt{x}} - 4$

2.2 13. The gross annual earnings of a certain company were $E(t) = 0.2t^2 + 9t + 30$ thousand dollars t years after its formation in 1990. At what percentage rate are the gross annual earnings growing with respect to time in 1995?

2.3 14. Differentiate and Simplify: $y = \frac{3x-1}{x^2+1}$

2.3 15. It is estimated that t years from now, the population of a certain suburban community will be $p(t) = 50 - \frac{7}{2t+1}$ thousand people. a) At what rate will the population be growing 3 years from now?
b) By how much will the population actually change during the 4th year?

2.5 16. The total cost of manufacturing q units of a certain commodity is $C(q) = 3q^2 + 8q + 9$.
a) Use marginal cost to estimate the cost of producing the 23rd unit.
b) Find the actual cost of producing the 23rd unit.

2.5 17. Suppose the total cost in dollars of manufacturing q units of a certain commodity is $C(q) = 3q^2 + 5q + 12$. If the current level of production is 30 units, estimate how the total cost will change if production is increased by one-half of a unit.

2.4 18. If $f(x) = \frac{1}{(3x^2-7x+5)^3}$, find $f'(x)$.

2.4 19. Differentiate and Simplify: $y = 5(x+2)^3(2x-3)^4$

2.4 20. The cost C of producing q units of a product is given by $C(q) = 4000 + 10q + 0.1q^2$. If the number of units as a function of the price per unit p is given by the equation $q(p) = 800 - 2.5p$, find the rate of change of cost with respect to price per unit when $p = 80$.

2.3 21. Find the second derivative, $f''(x)$ of the given function: $f(x) = \frac{3}{x} + \sqrt{x} - 5x^4 + 8x - 2$

3.1 22. Use calculus to find the intervals of increase and decrease of the function $f(x) = x^4 - 2x^3$

3.1 23. Let $f(x) = 2x^3 - 3x^2 - 12x + 13$. Find all first order critical points of f.

3.2 24. Determine where the graph of $f(x) = x^3 - 3x^2 - 9x + 1$ is concave up and concave down and give the coordinates of all inflection points.

3.2 25. Find the absolute maximum and minimum of the function $f(x) = 2x^3 - 7x^2 + 8x + 2$ on the interval $0 \leq x \leq 3$.

3.4 26. An efficiency study of the morning shift at a certain factory indicates that an average worker who arrives on the job at 8:00 am will have produced $Q(t) = -t^3 + 6t^2 + 18t$ units t hours later.
a) Compute the worker's rate of production at 9:00 a.m.
b) At what rate is the worker's rate of production changing with respect to time at 9:00 am?
c) At what time during the morning is the worker performing most efficiently?
d) At what time during the morning is the worker performing least efficiently?

3.4 27. The cost of producing a certain commodity is $C(x) = \frac{1}{8}x^2 + 3x + 98$ dollars when x units are produced. If the price of the commodity is $p(x) = \frac{1}{3}(75 - x)$ dollars per unit, use calculus to determine the level of production that maximizes profit.

3.5 28. The owner of an apple orchard estimates that if 24 trees are planted per acre, each mature tree will yield 600 apples a year. For each additional tree planted per acre the number of apples produced by each tree decreases by 12 a year. How many trees should be planted per acre to obtain the most apples per year?

4.1 29. If \$3000 is invested at 10% compounded continuously, what is the balance after 9 years?

4.2 30. Suppose the number of bacteria grows from 7000 to 10,000 in the first 15 minutes of an experiment. Assuming that the number of bacteria grows exponentially, how many bacteria will be present after 1 hour?

4.2 31. Solve for x: a) $\ln x = 3(\ln 2 - \ln 7)$ b) $2 + 3\ln x = 15$ c) $5 = 7 - 3e^{-2x}$

4.2 32. How long is it going to take for a certain amount of money to double if it is invested at 15% compounded continuously.

4.3 33. Find $f'(x)$ if: a) $f(x) = 2e^x$ b) $f(x) = 5\ln x$

4.3 34. Find $f'(x)$ if: a) $f(x) = 20 - 5e^{-.03x}$ b) $f(x) = \ln(x^2 + 4x - 3)$
c) $f(x) = \frac{\ln x^5}{x}$ d) $f(x) = x^2e^{3x}$

4.4 35. Find intervals of increase and decrease, study concavity, and sketch and label the graph of: $f(x) = 3 - 5e^{-x}$

4.4 36. Records indicate that t weeks after the outbreak of a disease, approximately $Q(t) = \frac{70}{3+52e^{-1.3t}}$ thousand people have been infected. At what rate was the disease spreading at the end of the third week?

5.1 37. Find the indicated integral:
a) $\int \left(\frac{1}{x^4} + x^{5/6} + x^2 - e^x - \frac{1}{x} + 7 + \sqrt{x} \right) dx$ b) $\int \frac{(3x+1)}{x^2}\, dx$

5.3 38. Evaluate the definite integral: $\int_0^1 (x^3 - 3x^2 + 5)\, dx$

5.3 39. Evaluate the definite integral: $\int_{-1}^3 (9x^2 - 30x + 25)\, dx$

5.4 40. Find the area of the region bounded by the curves $y = 1 + 4x - x^2$ and $y = 1 + x^2$

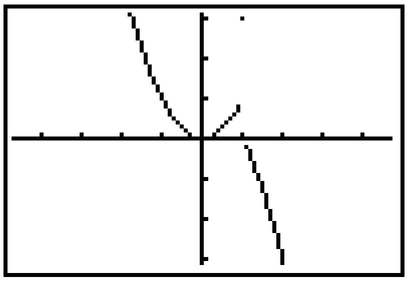
5.4 41. Find the area of the region bounded by the line $y = x$ and the curve $y = x^3$. (Sketch the graph of the area)

M119 — FINAL REVIEW — ANSWERS:

1) a) $x \neq -3, 3$ b) $x \geq \frac{4}{3}$

2) g(h(x)) = $\frac{x^2}{(x+1)^2} - \frac{x}{x+1} = \frac{-x}{(x+1)^2}$

3) C[q(t)] = $625t^2 + 25t + 900$, (b) C(3) = $6600, (c) after 4 hours

4)  note: the graph should have open endpoints at (1,1) and (1,0)

5) (0,0); (3,3); (–3,–3)

6) (a) y = $\frac{-6}{5}$x + 192 million gallons, (b) 183.6 million gallons

7)10 sleds

8) $ 70

9) a) $\frac{x(300\text{-}x^2)}{4}$ b) An x of 10 gives a maximum volume of 500 cm^3

10) $y = 2x + 2$.

11) (0,4)

12) f '(x) = $3x^2 + \frac{2}{x^2} + 3x^{1/2} + \frac{5}{2\sqrt{x^3}}$

13) 13.75%

14) y ' = $\frac{-3x^2 + 2x + 3}{(x^2+1)^2}$

15) a) .286 thousand, i.e.. 286 people, b) .222 thousand = 222 people

16) a) $C'(22) = 140$ b) $C(23) - C(22) = 143$

17) $dC = C'(30)\, dq$ = 185 (.5)=$92.50

18) f '(x) = $\frac{-18x + 21}{(3x^2 - 7x + 5)^4}$

19) y ' = $35(2x+1)(x+2)^2(2x-3)^3$

20) $C'(p) = 1.25p - 425$; $C'(80) = -325$

21) $f''(x) = \frac{6}{x^3} - \frac{1}{4\sqrt{x^3}} - 60x^2$

22) Increase: $x>\frac{3}{2}$ Decrease: $x<\frac{3}{2}$

23) (2,–7), (–1,20)

24) C.U $x > 1$ C.D. $x < 1$, inflection point $(1, -10)$

25) Ab Min (0,2) Abs Max (3,17)

26) a) 27 units per hour, b) 6 units per hour per hour c) 10 a.m., d) 8 a.m. and 12 noon

27) $x = 24$ Profit = Revenue − Cost = $xp - C(x) = -\frac{11}{24}x^2 + 22x - 98$

28) 37 trees (13 additional trees)

29) \$7378.81

30) $Q(60) = 29{,}154$

31-a) $\frac{8}{343}$

31-b) $e^{13/3} \approx 76.198$

31-c) $\frac{\ln(\frac{2}{3})}{-2} \approx .203$

32) 4.62 years

33-a) $f'(x) = 2e^x$

33-b) $f'(x) = \frac{5}{x}$

34-a) $f'(x) = .15e^{-.03x}$

34-b) $f'(x) = \frac{2x+4}{x^2+4x-3}$

34-c) $f'(x) = \frac{5-5\ln x}{x^2}$

34-d) $f'(x) = xe^{3x}(2 + 3x)$

35) always increasing, always concave down

36) $Q'(3) \approx 5832$ people per week

37-a) $-\frac{1}{3x^3} + \frac{6}{11}x^{11/6} + \frac{1}{3}x^3 - e^x - \ln|x| + 7x + \frac{2}{3}\sqrt{x^3} + C$

37-b) $3\ln|x| - \frac{1}{x} + C$

38) 17/4

39) 64

40) 8/3

41) 2(.25)= .5